Exam
SOLID MECHANICS (NASM)
November 2, 2022, 15:00-17:00 h

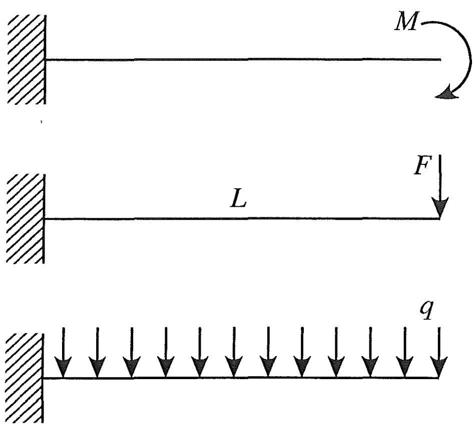
NB This is a *closed-book* exam, but students are allowed to have 1 page A4 (single-sided) as reference. The exam comprises four problems, for which one can obtain the following points:

Question	# points
1	1 + 1 = 2
2	3
3	2 + 3 + 2 = 7
4	2

The number of points is indicated next to each subquestion inside a rectangular box in the right-hand margin on the next pages.

The exam grade is calculated as $9*(\#\text{ points})/14+1)$ and contributes for 60% to the final grade for this course.

Reminder:

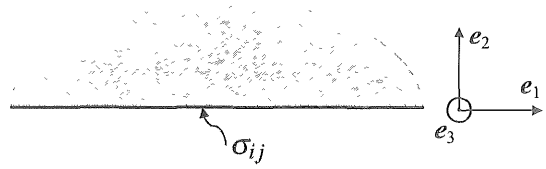


	$\theta(L)$	$w(L)$
Moment M	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$
Force F	$\frac{FL^2}{2EI}$	$\frac{FL^3}{3EI}$
Distributed load q	$\frac{qL^3}{6EI}$	$\frac{qL^4}{8EI}$

Figure 1: *Mysotis Mechanicus*: Tip deflections w and rotations θ of cantilever beams under end loading by a moment M, a force F or a distributed load of q per unit of length.

Question 1

a. Consider a flat interface between two dissimilar materials in a loaded composite material. When the system is in equilibrium, which of the stress components σ_{ij} in three dimensions are continuous across this interface (with unit normal $\boldsymbol{n} = \boldsymbol{e}_2$)? [1]



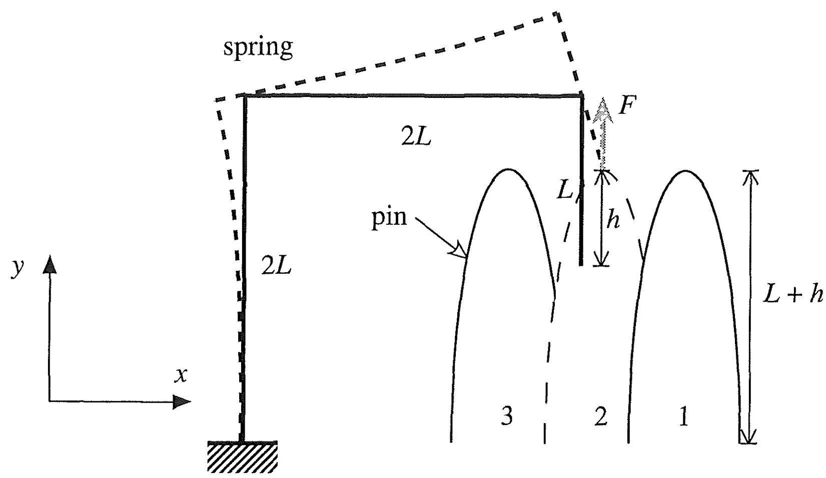
b. According to Mohr's circle, the three principal stresses $\sigma_1 > \sigma_2 > \sigma_3$ define three maximum shear stresses τ_1, τ_2, τ_3 in planes at 45° relative to subsequent sets of three principal stress directions. Can the hydrostatic stress be computed from these maximum shear stresses? [1]

Question 2 Consider a slender beam of length l and bending stiffness EI, and let it be subjected to a bending moment $M(x)$ $(0 \leq x \leq l)$. Show that the elastic bending energy can be written as

$$U = \int_0^l \frac{1}{2} M(x) w''(x) dx, \tag{1}$$

when $w(x)$ is the corresponding deflection. [3]

Question 3 A simple locking mechanism consists of a spring and a pin. The spring is L-shaped with two arms of length $2L$ and one of length L, and has bending modulus EI. The pin has a height $L+h$. As the pin slides from position 1 to 3 (or vice-versa), it will exert a force on the



spring that will make it bend upwards. The magnitude of this force depends on the shape of the pin, but here we only consider the most open position 2 indicated in the figure. When friction can be neglected, the pin at this position needs to bend the spring such that the tip displaces in the vertical y-direction by an amount h.

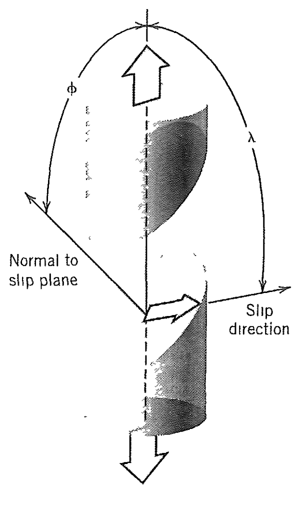
a. Compute the vertical force F in this state by application of the forget-me-nots (see first page, Fig. 1).[1] 2

b. In the absence of friction, the work done while going from position 1 to 2 (or from 3 to 2) is stored as elastic energy in the spring. In preparation of doing so, use Eq. (1) from the previous problem to express the bending energy in the cantilevers of the first two forget-me-nots in terms of the external loading M and F, respectively. 3

c. Compute the energy in the spring when the pin is in position 2. Discuss your result in the context of the work done while going from position 1 to 3 (or back). 2

Question 4 The figure on the right is copied from the section on slip in single crystals from Callister's textbook (fifth edition) used in your Materials Science course. In a crystal subjected to uniaxial tension, as indicated in the figure by the vertical arrows, a slip system is identified by the angle ϕ between the slip plane normal and the tensile direction, and by the angle λ between slip direction and tensile axis.

Derive the relation between the applied stress σ and the shear stress resolved on this slip system, τ_R.



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[1]Use linear theory, i.e. neglect second-order effects, and neglect deformation by uniaxial deformation.